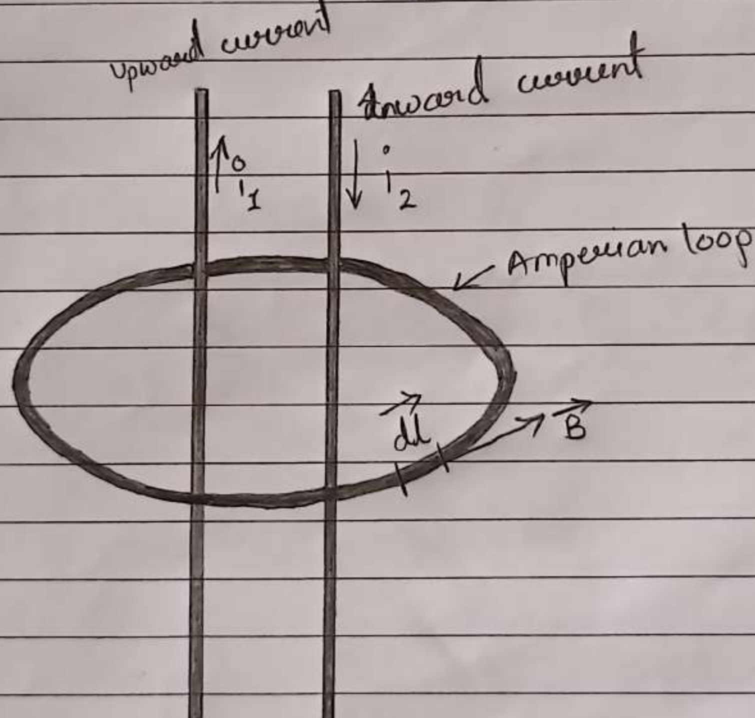
Ques 1 State Ampere's law & Explain the concept of displacement current.

Ans :-

Ampere's circuital law states that line integral of steady magnetic field over a closed loop is equal to μ_0 times the total current (I) passing through the surface bounded by the loop i.e.



$$\boxed{\oint \vec{B}.\vec{dl} = \mu_0 I_e}$$

I_e = Enclosed current.

The relation above involves a sign convention, given by right hand rule. let the fingers of the right hand be curled in the sense the boundary is traversed in the loop integral $\oint \vec{B}.\vec{dl}$. Then the direction of the thumb gives the sense in which the current I is regarded as positive. In the diagram shown i_1 is taken positive and i_2 is negative.

It should be similar to the gauss's law ampere's circuital law holds for any loop but it may not always facilitate an evaluation of the magnetic field in every case. It is best suited in which integral on the left side of the eq^n is solved easily using the symmetry of the situation.

For several applications, It is possible to choose the loop (called an Amperian loop) such that at each point on the loops, either

(i) $\vec{B}$ is tangential to the loop and has a non zero constant magnitude B, or

(ii) $\vec{B}$ is normal to the loop or

(iii) B vanishes everywhere on the loop.

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Need for Displacement current

Ampere's circuital law for conduction of current during charging of a capacitor was found inconsistent. Therefore, maxwell modified Ampere's circuital law by introducing the concept of displacement current

Displacement current

Displacement current is the current that is produced by the rate of change of the electric displacement field. It differs from the normal current that is produced by the motion of the electric charge, displacement current is the quantity explained in Maxwell's equation, it is measured in Ampere. Displacement current are produced by a time varying electric field rather than moving charges.

Defination.
A physical quantity related to Maxwell's equation that has the property of the electric current is called the displacement current. displacement current is defined as the rate of change of the electric displacement (D).

Current in Capacitor
A charging capacitor has no conduction of charge accumulation in the capacitor changes

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the electric field link with the capacitor that in turn produces the current called the displacement current.

$$\boxed{I_D = J_D S = S\frac{dD}{dt}}$$

where

S = area of the capacitor plate
I_D = Displacement current
J_D = Displacement current density
D = Electric field E.

$$\boxed{D = \varepsilon E}$$

ε = Permittivity of material between plates.

Displacement Current equations

Maxwell's eq^n define the displacement current which has the same unit as the electric current. The Maxwell field eq^n is represented as

$$\nabla \times H = J + J_D$$

H = magnetic field B as $B = \mu H$
μ = permeability of material b/w the plates
J = Conducting current density
J_D = displacement current density.

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we know that

$$\bar{\nabla}\cdot(\bar{\nabla}\times H) = 0$$

$$\bar{\nabla}\cdot J = -\frac{d\rho}{dt}$$

$$\bar{\nabla}\cdot J = -\bar{\nabla}\cdot \frac{dD}{dt}$$

using gauss Law

$$\nabla\cdot D = \rho$$

ρ= electric charge density

Thus ~~electric~~ displacement current density eqn is

$$\boxed{J_D = \frac{dD}{dt}}$$

Charactersitics of displacement current

In an electric circuit there are two types of current that are conduction current and the other is displacement current. Various characteristics of displacement current are mentioned below.

→ Displacement current does not appears from the actual movement of the electric charge as in the case of the conduction current but is produced by time changing electric field.

→ Displacement current is a vector quantity.

→ Electromagnetic wave propagate with the help of displacement current.

Ques 2 State and prove maxwell's equations.

Ans :- Maxwell first equation.

Maxwell's first equation is based on the Gauss law of electrostatic which states that "when a closed surface integral of electric flux density is always equal to charge enclosed over that surface."

Mathematically gauss law can be expressed as,

$$\oint \vec{D} \cdot d\vec{s} = Q_{enclosed} \quad -(1)$$

Any closed system will have multiple surfaces but a single volume, thus, the above surface integral can be converted into a volume integral by taking the divergence of the same vector, Thus mathematically it is

$$\oint \vec{D} \cdot d\vec{s} = \int \nabla \cdot \vec{D} \, d\vec{v} \quad -(2)$$

combining eq (1) and (2) we get

$$\int \nabla \cdot \vec{D} \, d\vec{v} = Q_{enclosed} \quad -(3)$$

volume charge density can be defined as-

$$\rho v = \frac{dQ}{dv}$$

$$dQ = \rho v dv$$

on integrating above eq^n we get

$$Q = \int \rho v dv \quad -(4)$$

substituting ④ in ③ we get

$$\int \nabla . D dv = \int \rho v dv$$

$$\boxed{\nabla . D dv = \rho v}$$

This is required 1st maxwell equation.

Maxwell Second Equation

Maxwell second equation is based on gauss law on magnetostatics.

Gauss law on magnetostatics states that "a closed surface integral of magnetic flux density is always equal to total scalar magnetic flux enclosed within that surface of any shape or size lying in any medium."

$$\oint \vec{B} . ds = \phi_{enclosed} \quad -①$$

Hence we can conclude that magnetic flux cannot be enclosed within a closed surface of any shape.

$$\oint \vec{B} . ds = 0 \quad -②$$

Applying the gauss divergence theorem to eqn ② we can convert it into volume integral by taking the divergence of the same vector.

$$\oint \vec{B} . ds = \int \nabla . \vec{B} dv \quad -③$$

Substituting eqn ③ in ② we get -

$\int \nabla \cdot \vec{B} \, dv = 0 \quad -(4)$

Here to satisfy the above eqn either

$\int dv = 0$

or $\nabla \cdot \vec{B} = 0$

The volume of any body or object can never be zero

Thus we arrive at maxwell's second equation.

$$\boxed{\nabla \cdot \vec{B} = 0}$$

Where $\vec{B} = \mu \vec{H}$ ... is flux density

Maxwell Third Equation

Maxwell's 3rd eqn is derived from faraday's laws of Electromagnetic induction. it states that "Whenever there are n- turns of conducting coil in a closed path placed in a time-varying magnetic field, an alternating electromotive force gets induced in each coil."

Lenz's law gives this. which states, "An induced electromotive force always opposes the time-varying magnetic flux."

Mathematically it is expressed as-

Alternating emf,

$emf_{alt} = -N \frac{d\phi}{dt} \quad -(1)$

N= no of turns in a coil.

ϕ = scalar magnetic flux

Let $N = 1$

$$emf_{alt} = -\frac{d\phi}{dt} \quad —②$$

here scalar magnetic flux can be replaced by -

$$\phi = \int \vec{B}\cdot d\vec{s} \quad —③$$

Substitute eqⁿ ③ in ②.

$$emf_{alt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

which is partial D.E. given by -

$$emf_{alt} = \int -\frac{d\vec{B}}{dt}\cdot d\vec{s} \quad —④$$

The alternating electromotive force induced in a coil is basically a closed path.

$$emf_{alt} = \oint \vec{E}\cdot d\vec{l} \quad —⑤$$

Substituting eqⁿ ⑤ in ④ we get -

$$\oint \vec{E}\, d\vec{l} = \int -\frac{\partial \vec{B}}{\partial t}\cdot ds \quad —⑥$$

$$\oint \vec{E}\, d\vec{l} = \int (\nabla \times \vec{E})\cdot d\vec{s} \quad —⑦$$

(using stoke's theorem)

Substituting eqⁿ ⑦ in ⑥ we get

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$$\int (\nabla \times \vec{E}) d\vec{s} = \int -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad \text{--- (8)}$$

The surface integral can be canceled on both sides and we get

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

This is required 3^{rd} maxwell's equation.

Maxwell's Fourth Equation.

Maxwell's fourth equation is derived from Ampere's law which states that "Magnetic field can be either produced by electric current or by the altering electric field". The magnetic field vector's closed line integral is equal to the total quantity of scalar electric field present in the path of that shape. This Maxwell's eqn also defines the displacement current. The electric current and displacement current through a closed surface is directly proportional to the induced magnetic field around any closed loop.

Maxwell's added the displacement current to Ampere's law. Mathematical representation of maxwell's equation Fourth Equation.

Closed line integral of magnetic field vector = Total quantity of scalar electric field present.

$$\oint \vec{H}\cdot\vec{dl} = I \quad -①$$

$$\oint \vec{H}\cdot\vec{dl} = \int(\nabla\times\vec{H})\cdot ds \quad -②$$

(by stoke's theorem)

using ② in eq ①

$$\int(\nabla\times\vec{H})\cdot\vec{dl} = I \quad -③$$

$\int(\nabla\times\vec{H})\cdot\vec{dl}$ = Vector quantity.

I = scalar quantity.

Multiply I by density vector.

$$\vec{J} = \frac{I}{s}\hat{a}N$$

$\vec{J}$ = Difference in scalar electric field / Difference in vector electric field $\vec{J}$.

$$\frac{dl}{ds.dl} = \vec{J}\cdot ds$$

$$I = \int\vec{J}\cdot\vec{ds} \quad -④$$

Using eq ④ in eq ③.

$$\int(\nabla\times\vec{H})\cdot\vec{dl} = \int\vec{J}\cdot\vec{ds} \quad -⑤$$

Cancelling the surface integral from both sides, we get maxwell's fourth equation

$$\boxed{\vec{J} = \nabla\times H}$$

$$\boxed{(\nabla \times \vec{H}) = \vec{J} + \vec{J}_D}$$

Ques 3 State and prove poynting theorem.

Ans:-

Statement :- This theorem states that the cross product of electric field vector, E and magnetic field vector, H at any point is a measure of the rate of flow of electromagnetic energy per unit area at that point, that is

$$P = E \times H$$

P = Poynting vector, $P \perp E$ and H

Proof :-

Consider maxwell's fourth eq^n that is

$$\text{del} \times H = J + \varepsilon \frac{dE}{dt}$$

or $J = (\text{del} \times H) - \varepsilon \frac{dE}{dt}$

$$E \cdot J = E \cdot (\text{del} \times H) - \varepsilon E \cdot \frac{dE}{dt} \quad \text{— (1)}$$

(taking dot product with E)

we vector identity

$$\text{del} \cdot (E \times H) = H \cdot (\text{del} \times E) - E \cdot (\text{del} \times H)$$

or $E \cdot (\text{del} \times H) = H \cdot (\text{del} \times E) - \text{del} (E \times H)$

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By substituting value of E.(del × H) in eqⁿ ① we get

$$E \cdot J = H \cdot (\text{del} \times E) - \text{del} \cdot (E \times H) - \varepsilon E \frac{dE}{dt} \quad -②$$

also from Maxwell's third eqⁿ (Faraday's law of electromagnetic induction).

$$\text{del} \times E = \mu \frac{dH}{dt}$$

by substituting value of del×E in eqⁿ ② we get

$$E \cdot J = \mu H \cdot \left(\frac{dH}{dt}\right) - \varepsilon E \cdot \frac{dE}{dt} - \text{del} \cdot (E \times H) \quad -③$$

we can write

$$H \cdot \frac{dH}{dt} = \frac{1}{2} \frac{dH^2}{dt} \quad - 4a$$

$$E \cdot \frac{dE}{dt} = \frac{1}{2} \frac{dE^2}{dt} \quad - 4b$$

By substituting eqⁿ 4a and 4b in eqⁿ 3 we get.

$$E.J. = -\frac{\mu}{2} \frac{dH^2}{dt} - \frac{\varepsilon}{2} \frac{dE^2}{dt} - \text{del} \cdot (E \times H)$$

$$E \cdot J = -\frac{d}{dt}\left(\frac{dH^2}{2} + \frac{\varepsilon E^2}{2}\right) - \text{del} \cdot (E \times H)$$

By taking volume integral on both sides we get.

$$\int E.J.\,dV = -\frac{d}{dt}\int\left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right)dv - \int del.(E\times H)\,dV \quad -(5)$$

apply gauss's divergence theorem to second term of R.H.S. to change volume integral into surface integral, that is

$$\int del.(E\times H)\,dV = \int (E\times H).ds$$

Substitute above eq^n in eq^n (5)

Thus

$$\int E.J.\,dV = -\frac{d}{dt}\int\left[\varepsilon E^2/2 + \mu H^2/2\right]dv - \int (E\times H).ds \quad -(6)$$

or

$$\int (E\times H).ds = \int -\frac{d}{dt}\left[\varepsilon E^2/2 + \mu H^2/2\right]dv - \int E.J.dV$$

$$\boxed{\int (E\times H)ds = \int -\frac{d}{dt}\left[\varepsilon E^2/2 + \mu H^2/2\right]dv - \int E.J.dV}$$

Hence proved

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Qs 4 Explain schrodinger's wave function and its characteristics.

Ans:- Schrodinger's wave function, denoted by the symbol ψ is a mathematical expression that describes the quantum state of a system of one or more particles. It can be used to calculate the probability of finding a particle in a given region of space and time, as well as other physical properties such as energy, momentum, angular momentum, etc.

The wave function is a solution of the schrodinger equation which is a partial differential equation that governs the evolution of quantum systems. The Schrodinger equation can be written as:

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi}$$

i = imaginary unit
$\hbar$ = reduced plank constant.
ψ = wave function
t = time
$\hat{H}$ = Hamiltonian operator, which represents the total energy of the system.

The wave function has the following important characteristics :

→ It is a complex-valued function, meaning that it can have both real and imaginary parts. The complex nature of the wave function reflects the phase of the matter wave associated with the particle.

→ It is normalized, meaning that the integral of its squared modulus over all space is equal to one. This ensures that the total probability of finding the particle somewhere in the universe is 100%. Mathematically, this condition can be expressed as:

$$\boxed{\int_{-\infty}^{\infty} |\psi|^2 \, dx = 1}$$

where $|\psi|^2$ is the squared modulus of the wave function, and dx is the infinitesimal element of space.

→ It is continuous and single-valued, meaning that it does not have any breaks or jumps, and that it does not have multiple values for the same point in space and time. These conditions ensure that the wave function is well-defined and physically meaningful.

→ It is orthonormal, meaning that the inner product of two different wave functions is zero, and the inner product of a wave function with itself is one. This property allows us to use the wave function as a basis for representing any quantum states as a linear combination of wave functions. Mathematically, this condition can be expressed as:

$$\boxed{\langle \psi_i | \psi_j \rangle = \delta_{ij}}$$

$\psi_i | \psi_j$ = Inner product of wave functions ψ_i and ψ_j and δ_{ij} is the Kronecker delta, which is one if $i = j$ and zero otherwise.

Ques 5 Derive Schrodinger's time dependent and time independent wave equations.

Ans:

Time dependent Schrodinger's Equation :- Consider a particle of mass m moving in +ve x-direction ~~it~~ P. in a conservative force field, The potential energy of the particle is $V(x)$, momentum in X-direction is P_x and the total energy of the particle is E. The matter wave associated with the particle can be expressed by a one-dimensional wave function as follows:

$$\psi = ae^{-i(Et - P_x x)/h} \quad \text{or} \quad \psi = ae^{i(P_x x - Et)/h} \qquad -(1)$$

Partially differentiating the eq^n (1) with respect to time t.

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{h} ae^{i(P_x x - Et)/h} = -\frac{iE}{h} \psi$$

or $$E\psi = ih \frac{\partial \psi}{\partial t} \qquad -(2)$$

Similarly partially differentiating eq^n (1) w.r.to x,

$$\frac{\partial \psi}{\partial x} = \frac{iP_x}{h} ae^{i(P_x x - Et)/h} = \frac{iP_x}{h} \psi$$

or $$P_x \psi = -ih \frac{\partial \psi}{\partial x} \qquad -(3)$$

According to classical mechanics, the total energy of the particle

$$\boxed{E = \frac{P_x^2}{2m} + V}$$

or $$E\psi = \left(\frac{P_x^2}{2m} + V\right)\psi$$

Substituting the values of E and P_x from eqⁿs ② and ③ we get.

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{1}{2m}(-i\hbar)^2 \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

or $$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi \quad \text{— (4)}$$

This is the time dependent Schrodinger's wave eqⁿ for a one dimensional moving particle under a conservative force. The solution $\psi(x,t)$ of this eqⁿ is called the time dependent wave function.

For free particle, V=0 hence $i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}$ — (5)

from eqⁿ (4), in three dimensional case,

$$\boxed{\frac{-\hbar^2}{2m} \nabla^2 \psi + V\psi = i\hbar \frac{\partial \psi}{\partial t}} \quad \text{— (6)}$$

This equation is the time dependent Schrodinger's equation.

Time independent Schrodinger's Equation

Sometimes we come across situations where the potential energy of the particle does not depend on time t and the potential energy is only the function of position. In such cases, the behaviour of particle is expressed by the time independent schrodinger equation.

Consider a particle of mass m moving in positive x-direction under a force $F(x)$. In this force field, the potential energy of the particle V does not depend on the time t, but depends only on the position x (i.e. $V = V(x)$). Let momentum of particle in x-direction be P_x and the total energy of particle be E.

The wave representing the plane wave associated with the particle is

$$\psi = ae^{-i(Et - P_x x)/h} \quad \text{or} \quad \psi = ae^{i(P_x x - Et)/h} \qquad \text{--- (1)}$$

Partially differenting eqn (1) with respect to x, we get

$$\frac{\partial \psi}{\partial x} = \frac{iP_x}{h} ae^{i(P_x x - Et)/h} = \frac{iP_x}{h}\psi \quad \text{or}$$

$$P_x \psi = -ih\frac{\partial \psi}{\partial x} \qquad \text{--- (2)}$$

According to classical mechanics, total energy of particle

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$$E = \frac{P_x^2}{2m} + V$$

or $$E\psi = \left(\frac{P_x^2}{2m} + V\right)\psi$$

substituting the value of P_x from eqⁿ ②, we get

$$E\psi = \frac{1}{2m}(-i\hbar)^2 \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

or $$E\psi = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi$$

or $$\frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi \text{ or } \frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{— (3)}$$

This is the time independent schrodinger's eqn for one-dimensional motion of a particle under the conservative force field. The solⁿ $\psi_{(x)}$ is called the time independent wave function.

For the free particle $V=0$,

hence $$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}E\psi = 0 \quad \text{— (4)}$$

from eq ③ in three dimensional case,

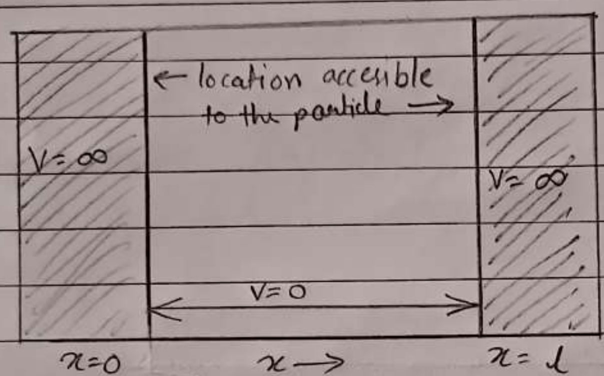
The time independent schrodinger's wave eqn will be.

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad — \ (5)}$$

Ques 6 Solve schrodinger's wave eqn for the free particle in a potential box.

Ans:-

The particle in a box model is diagrammed in figure



Potential energy vs distance for a particle in a box.

The potential energy constraints mean that the amplitude of the particle's wavefunction must be zero, $\psi(x)=0$, when the value of x lies in the interval

$$-\infty x < 0$$

or

$$l < x < +\infty$$

we assume that the probability of finding the particle cannot change abruptly when its location changes by an arbitrarily small amount. This means that the wavefunction must be continuous, and it follows that $\psi(0)=0$ and $\psi(l)=0$. Inside the box, the particle's Schrodinger equation is

$$\left(\frac{-h^2}{8\pi^2 m}\right)\frac{d^2\psi}{dx^2} = E\psi$$

and we seek those function $\psi(x)$ that satisfy both this differential eqn and the constraint eqns $\psi(0) = 0$ and $\psi(l) = 0$. It turns out that there are infinitely many such solutions, ψ_n, each of which corresponds to a unique energy level, E_n.

To find these solutions, we first ~~guided~~ guess - guided by our considerations in ~~§2~~ §2 - that solutions will be of the form

$$\psi(x) = A\sin(ax) + B\cos(bx)$$

A solution must satisfy

$$\psi(0) = A\sin(0) + B\cos(0) = B\cos(0) = 0$$

So that $B=0$. At the other end of the box, we must have

$$\psi(l) = A\sin(al) = 0$$

which means that $al = n\pi$, where n is any integer: $n = 1, 2, \ldots$ hence we have

$$a = n\pi / l$$

and the only eqn e of the proposed from that satisfy the conditions at the ends of the box are

$$\psi_n(x) = A\sin(n\pi x/l)$$

To test whether these eq^ns satisfy the schrodinger equation, we check

$$\left(\frac{-h^2}{8\pi^2 m}\right)\frac{d^2}{dx^2}\left[A\sin\left(\frac{n\pi x}{l}\right)\right] = E_n \psi_n$$

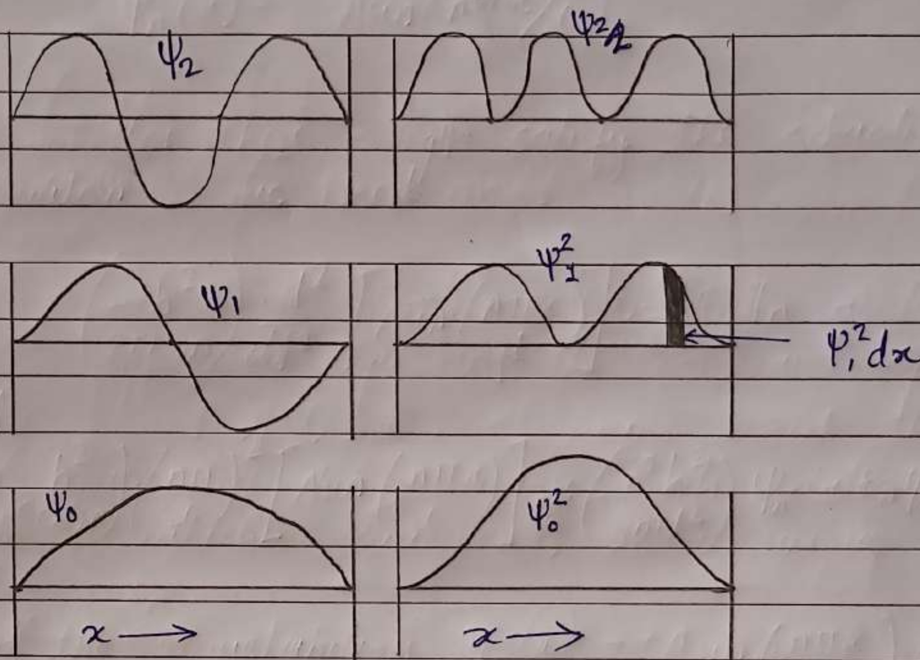
and find

$$\left(\frac{h^2}{8\pi^2 m}\right)\left(\frac{n\pi}{l}\right)^2\left[A\sin\left(\frac{n\pi x}{l}\right)\right] = \left(\frac{n^2 h^2}{8ml^2}\right)\psi_n = E_n\psi_n$$

So that the wavefunctions $\psi_n(x) = A\sin\left(\frac{n\pi x}{l}\right)$ are indeed solutions and the energy, E_n, associated with the wavefunction $\psi_n(x)$ is

$$E_n = \frac{n^2 h^2}{8ml^2}$$

We see that the energy values are quantized; although there are infinitely many energy levels, E_n, only very particular real numbers - those given by the equation above - correspond to energies that the particle can have. If we sketch the first few wavefunctions, $\psi_n(x)$, we see that there are always $n-1$ locations inside the box at which $\psi_n(x)$ is zero. These locations are called nodes. Once we known, we know the no of nodes and we can sketch the general shape of the corresponding wave function. The first three wavefunctions and their squares are sketched in figure.



wave f^n of particle in a box.

To determine A, we interpret $\psi^2(x)$ as a probability density f^n, and we require that the probability of finding the particle in the box be equal to unity. This means that

$$1 = \int_0^l A^2 \sin^2\left(\frac{n\pi x}{l}\right)dx = A^2\int_0^l\left[\frac{1}{2} - \frac{1}{2}\cos\left(\frac{2n\pi x}{l}\right)\right]dx = A^2\left(\frac{l}{2}\right)$$

so that $A = \sqrt{2/l}$, and the final wave functions are

$$\boxed{\psi_n(x) = \sqrt{\frac{2}{l}}\,\sin\left(\frac{n\pi x}{l}\right)}$$